

Finite Size Effects in Thermal Field Theory

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Abstract

We consider a neutral self-interacting massive scalar field defined in a d -dimensional Euclidean space. Assuming thermal equilibrium, we discuss the one-loop perturbative renormalization of this theory in the presence of rigid boundary surfaces (two parallel hyperplanes), which break translational symmetry. In order to identify the singular parts of the one-loop two-point and four-point Schwinger functions, we use a combination of dimensional and zeta-function analytic regularization procedures. The infinities which occur in both the regularized one-loop two-point and four-point Schwinger functions fall into two distinct classes: local divergences that could be renormalized with the introduction of the usual bulk counterterms, and surface divergences that demand counterterms concentrated on the boundaries. We present the detailed form of the surface divergences and discuss different strategies that one can assume to solve the problem of the surface divergences. We also briefly mention how to overcome the difficulties generated by

infrared divergences in the case of Neumann-Neumann boundary conditions.

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1 Introduction

The Casimir effect is the manifestation of the zero-point energy of the quantized electromagnetic field, in the presence of metallic plates [1]. A very simple calculation predicts that in a four-dimensional spacetime, uncharged perfectly conducting parallel plates should attract each other with a force per unit area $F(L) \propto \frac{1}{L^4}$, where L is the distance between the plates. Extensive reviews of this subject can be found in Refs. [2] [3] [4] [5] [6]. As stressed by Milloni et al. [7], a brief argument showing that the zero point-energy associated with the quantized electromagnetic field must have a physical meaning was already given by Einstein and Stern [8]. These authors noted that a zero-point energy seems necessary in order to avoid a first-order quantum correction to β^{-1} in the classical limit $\beta \gg \omega$ in Planck's expression for the average energy of an oscillator in equilibrium with radiation at temperature β^{-1} .

Although the vacuum energies of different physical configurations are formally divergent, their difference can be finite. In the case of a free scalar field, interacting only with boundary surfaces, the Casimir approach can be summarized as follows: first a complete set of modes solutions of the Klein-Gordon equation satisfying appropriate boundary conditions, and their respective eigenfrequencies are presented. Next, the divergent zero-point energy is regularized by the introduction of an ultraviolet cut-off. Finally, the polar part of the regularized energy is removed using a renormalization procedure. This procedure was first discussed by Fierz [9] a long time ago, followed by Boyer [10] and also by Svaiter and Svaiter [11] [12]. In these two last references, an attempt to clarify the relation between the cut-off method and analytic regularization procedures in Casimir

effect has been developed. In particular, in these papers an analytic regularization procedure was interpreted as a cut-off method, and using a mixed cut-off in the regularized zero-point energy, it was possible to unify these two methods both in two- and three-dimensional spacetimes. Further, a general proof was given that when the introduction of an exponential cut-off yields an analytic function with a pole at the origin, then the analytic regularization using the zeta function (or a generalization for the zeta function) is equivalent to the application of a cut-off with the subtraction of the singular part at the origin [13] [14]. More recently, Fulling offered an interesting discussion with regard to the problems in the renormalization program devised to find the renormalized vacuum stress-tensor in different field theories [15].

It is important to point out that these results are valid at one-loop level and one is dealing with free fields only. It is clear that the formalism must be generalized to take into account the case of self-interacting fields. Although higher-loop corrections to the Casimir effect seem beyond experimental reach today, theoretically such corrections are certainly of interest. Nevertheless, with the exception for some few papers, only global issues have been discussed in the study of radiative corrections to the Casimir effect. One such exception is the discussion presented by Robaschik et al. [16]. With this scenario in mind, it is natural to ask the important question: how to implement the perturbative renormalization algorithm, assuming the presence of rigid boundaries (hard-walls), using the standard weak-coupling perturbative expansion in quantum field theory, that is, how to implement the one-loop perturbative renormalization of a self-interacting scalar theory, assuming boundary conditions which do break translational symmetry. Our aim when studying these issues

is linked to the following question: does the infrared problem have a solution in theories where translational invariance is broken? Note that temperature effects can solve the infrared problem in some models in quantum field theory [17]; for a recent treatment in non-abelian gauge theories at high temperature, and the infrared problem, see for example Ref. [18]. Also, in massless scalar $\lambda\varphi^4$ theory, if thermal equilibrium with a reservoir is assumed, the infrared problem can be solved after a resummation procedure. The standard is to use the Dyson-Schwinger equation to write a non-perturbative version of the self-energy gap equation, or to use the composite operator formalism [19] [20] [21].

We would like to call the attention of the reader that there are some disagreements in the literature as to implementing the one-loop perturbative renormalization in finite size systems when translational invariance is broken. In the one-loop approximation, Albuquerque et al. [22] found that the mass counterterm depends on the size of the compact dimension in the $\lambda\varphi^4$ theory. Also, Malbouisson et al. [23] assumed a self-interacting scalar field confined between two infinite parallel plates, and using the techniques developed by Ananos et al. [21] these authors didn't find any surface counterterm in the $\lambda\varphi^4$ theory at finite temperature. Furthermore, they were able to define temperature and size-dependent mass and coupling constant terms in systems where translational invariance is broken.

The purpose of this paper is to present a detailed calculation of the one-loop renormalization of the $\lambda\varphi^4$ theory at finite temperature, assuming that one of the spatial coordinates is confined to a finite interval. Since this assumption is not sufficient to explicitly breaking the translational sym-

metry, we will further introduce boundary surfaces where the field satisfies appropriate boundary conditions. In this situation, the breaking of the translational invariance of the theory is ensured. This paper is a natural continuation of the papers of Fosco and Svaiter [24] and also Caicedo and Svaiter [25]. Our aim is to further the understanding of the renormalization procedure in systems at finite temperature where there is a break of translational symmetry. We will discuss the Dirichlet-Dirichlet (DD) and also the Neumann-Neumann (NN) boundary conditions. For the Dirichlet-Dirichlet boundary conditions, the model is free of infrared divergences. In the Neumann-Neumann boundary conditions case, infrared divergences associated with zero modes will appear for bare massless fields. We show that there is no clear meaning for a thermal- or size-dependent mass in such situations. Consequently, can not be used to solve the infrared problem in the case of Neumann-Neumann boundary conditions a resummation procedure.

The organization of the paper is the following: in the section II we sketch the general formalism of the theory, deriving the one-loop two-point and four-point functions. In section III we use two different analytic regularization procedures, i.e, dimensional regularization and zeta-function analytic regularization, to identify the polar contributions that appear in the expressions of the one-loop two-point and four-point Schwinger functions. In section IV we renormalize the four-point Schwinger function and the problem for the infrared divergences is raised. In the conclusions we will discuss alternative solutions for the problem of the surfaces counterterms. In this paper we use $\hbar = c = k_B = 1$.

2 General Formalism and the Finite Temperature Generating Functional of Schwinger Functions

The static properties of finite temperature field theory can be derived from the partition function [26]. To obtain the partition function the starting point is the Feynman, Matheus and Salam approach [27]. Thus, let us consider the generating functional of (complete) Green's functions for a self-interacting scalar field theory defined in a flat d -dimensional Euclidean space $Z(h)$, given by

$$Z(h) = \int [d\varphi] \exp \left(-S[\varphi] + \int d^d x h(x) \varphi(x) \right), \quad (1)$$

where $[d\varphi]$ is a translational invariant measure (formally given by $[d\varphi] = \prod_{x \in R^d} d\varphi(x)$) and $S[\varphi]$ is the classical action associated with the scalar field. The quantity $Z(h)$ can be regarded as the functional integral representation for the imaginary time evolution operator $\langle \varphi_2 | U(t_2, t_1) | \varphi_1 \rangle$, with boundary conditions $\varphi(t_1, \vec{x}) = \varphi_1(\vec{x})$ and $\varphi(t_2, \vec{x}) = \varphi_2(\vec{x})$ which gives the transition amplitude from the initial state $|\varphi_1\rangle$ to a final state $|\varphi_2\rangle$ in the presence of some scalar source of compact support. As usual, the generating functional of the connected correlation functions shall be given by $W(h) = \ln Z(h)$. In a free scalar theory, $Z(h)$ as well as $W(h)$ can be calculated exactly. Regarding the Lagrangian density, we assume that

$$\mathcal{L}(\varphi, \partial\varphi) = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m_0^2\varphi^2 + \frac{1}{4!}\lambda_0\varphi^4, \quad (2)$$

where m_0 is the bare mass and λ_0 is the bare coupling constant of the model. We are also assuming $m_0^2 \geq 0$ and also $\lambda_0 > 0$. The Euclidean n -point correlation functions, i.e., the n -point Schwinger

functions are given by the expectation value with respect to the weight $\exp(-S(\varphi))$, defined as

$$G^{(n)}(x_1, x_2, \dots, x_n) = \frac{1}{Z(h)} \frac{\delta^n Z(h)}{\delta h(x_1) \dots \delta h(x_n)} \Big|_{h=0}. \quad (3)$$

The n -point connected correlation functions $G_c^{(n)}(x_1, x_2, \dots, x_n)$ are given by

$$G_c^{(n)}(x_1, x_2, \dots, x_n) = \frac{\delta^n W(h)}{\delta h(x_1) \dots \delta h(x_n)} \Big|_{h=0}. \quad (4)$$

Finally, the generating functional of connected one-particle irreducible correlation functions (the effective action) is introduced by performing a Legendre transformation on $W(h)$,

$$\Gamma(\varphi_0) = -W(h) + \int d^d x \varphi_0(x) h(x). \quad (5)$$

Let us define the proper vertices $\Gamma^{(n)}(x_1, \dots, x_n)$ as:

$$\Gamma^{(n)}(x_1, \dots, x_n) = \frac{\delta^n \Gamma(\varphi_0)}{\delta \varphi_0(x_1) \dots \delta \varphi_0(x_n)} \Big|_{\varphi_0=0}, \quad (6)$$

where the normalized vacuum expectation value of the field $\varphi_0(x)$ is given by

$$\varphi_0(x) = \frac{\delta W}{\delta h(x)}. \quad (7)$$

It is clear that in the case of a single scalar field, for a zero normalized vacuum expectation value of the field $\varphi_0(x)$, the effective action may be represented as a functional power series around the value $\varphi_0 = 0$, with the form

$$\Gamma(\varphi_0) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^d x_1 \dots d^d x_n \Gamma^{(n)}(x_1, \dots, x_n) \varphi_0(x_1) \dots \varphi_0(x_n). \quad (8)$$

If the bare coupling constant vanishes, i.e., $\lambda_0 = 0$, the generating functional of all n -point Schwinger functions $Z(h)$ can be calculated exactly, since we have to evaluate only Gaussian integrals. After some manipulations we obtain that the Gaussian generating functional $Z_0(h)$ is given by

$$Z_0(h) = \exp \left(\frac{1}{2} \int d^d x \int d^d y h(x) G_0^{(2)}(x-y, m_0) h(y) \right), \quad (9)$$

where the two-point Schwinger function (the inverse kernel) satisfies

$$(-\Delta_x + m_0^2) G_0^{(2)}(x-y, m_0) = \delta^d(x-y). \quad (10)$$

In this situation, the free Euclidean field is a gaussian random variable defined by its two-point correlation function

$$G_0^{(2)}(x-y, m_0) = \langle x | (-\Delta + m_0^2)^{-1} | y \rangle, \quad (11)$$

and the two-point Schwinger function has a well known Fourier representation given by

$$G_0^{(2)}(x-y, m_0) = \frac{1}{(2\pi)^d} \int d^d p \frac{e^{ip(x-y)}}{(p^2 + m_0^2)}. \quad (12)$$

In the next chapter we will show that the two-point function $G_0^{(2)}(x-y, m_0)$ can be expressed in terms of the modified Bessel function of the third kind or Macdonald's function $K_\mu(x)$. At present, we are not interested in evaluating the two-point Schwinger function, but only in the analysis of the behavior of $G_0^{(2)}(x-y, m_0)$ in a given ϵ -neighborhood. Let us assume that $m|x-y| < 1$; in this case, for $d \geq 3$ we can use that $G_0^{(2)}(x-y, m_0^2) \approx G_0^{(2)}(x-y, m_0^2 = 0) = |x-y|^{-(d-2)}$. For $d = 2$, the mass parameter can not be eliminated from the denominator and we have the following

short distance behavior: $G_0^{(2)}(x - y, m_0^2) \propto \ln(m|x - y|)$. It is well known that a massless two-dimensional scalar field theory is not consistent, once the model has severe infrared divergences. There are different proposals to circumvent this problem; we only mention some of them. For instance, one may violate the positivity of the state vector space; another attempt is to restrict the test functions of the theory, and finally one can introduce a cut-off in the definition of the positive and negative Wightman functions. It is clear that such cut-off procedure is equivalent to introducing a box to regulate the theory in the infrared. Later, we will discuss other strategies to solve the problem of the infrared divergences in scalar theories at finite temperature.

Coming back to the generating functional of all Schwinger functions, for $\lambda_0 \neq 0$ it is not possible to find a closed exact expression for the partition function, and a perturbative expansion is mandatory. Let us then assume the weak-coupling perturbative expansion of the theory. It is important to point out that the partition function can be defined in arbitrary geometries, and classical boundary conditions must be implemented in the two-point Schwinger function, restricting the space of functions that appear in the functional integrals. If we want to include thermal effects, and assuming thermal equilibrium, from the Feynman, Matheus and Salam formula we have:

$$\langle \varphi_b | e^{-iH(t_f - t_i)} | \varphi_a \rangle = \int_{\varphi(t_i) = \varphi_a}^{\varphi(t_f) = \varphi_b} \exp \left(i \int_{t_i}^{t_f} dt \int d^{d-1}x \mathcal{L}(\varphi, \partial\varphi) \right), \quad (13)$$

where we have to assume that $t_f - t_i = -i\beta$ and also set $\varphi_a = \varphi_b$, and the sum over all φ_a must be performed in order to produce the trace. The partition function $Tr [e^{-\beta H}]$ is given by

$$Tr [e^{-\beta H}] = \int_{periodic} [d\varphi] \exp \left(i \int_{t_i}^{t_i - i\beta} dt \int d^{d-1}x \mathcal{L}(\varphi, \partial\varphi) \right), \quad (14)$$

where the integration over the fields satisfying $\varphi(t_i - i\beta, \vec{x}) = \varphi(t, \vec{x})$. Since the time integration must range from some value t_i to $t_i - i\beta$, let $t_i = 0$ and set the contour along the negative imaginary axis from 0 to $-i\beta$. Thus, $t = -i\tau$, where $0 \leq \tau \leq \beta$, and we have

$$Z(h)|_{h=0} = \int_{periodic} [d\varphi] \exp \left(\int_0^\beta d\tau \int d^{d-1}x \mathcal{L}(\varphi, \partial\varphi) \right). \quad (15)$$

To generate the n -point Schwinger functions we need to couple the field with an external source. We will assume that the system is confined between two parallel hyperplanes, (which we call the Casimir configuration), localized at $z = 0$ and $z = L$, and we are using cartesian coordinates $x^\mu = (\vec{r}, z)$, where \vec{r} is a $(d - 1)$ dimensional vector perpendicular to the \vec{z} vector. Note that since we assume thermal equilibrium with a reservoir, we have periodicity in the first coordinate and $0 \leq r_1 \leq \beta$. See for example Ref. [28], or for a complete review of quantum field theory at thermal equilibrium, see for example Ref. [29]. The choice of Dirichlet-Dirichlet boundary conditions means that the scalar field satisfies

$$\varphi(\vec{r}, z)|_{z=0} = \varphi(\vec{r}, z)|_{z=L}, \quad (16)$$

and Neumann-Neumann boundary conditions means that

$$\frac{\partial}{\partial z} \varphi(\vec{r}, z)|_{z=0} = \frac{\partial}{\partial z} \varphi(\vec{r}, z)|_{z=L}. \quad (17)$$

In the next section we will discuss the perturbative renormalization at the one-loop level of the field theory in the presence of rigid boundaries. The great interest of this matter is: when systems contain macroscopic structures, how it is possible to implement the renormalization program?

We will examine how does the weak-coupling perturbative expansion and the renormalization program can be implemented. In order to identify the singular part of the one-loop two-point Schwinger function, we use a combination of dimensional and zeta-function analytic regularization procedures. We also present the detailed form of the surface divergences. Note that due to our choice (two-parallel hyperplates), the region outside the boundaries is the union of two-simple connected domains. The renormalization of the field theory in such exterior regions must be carried out along the same lines as for the interior region. For simplicity we are considering only the interior region.

3 The regularized one-loop two and four-point Schwinger functions

The aim of this section is to reshape a well known result, adding finite temperature effects to the problem. In order to implement the renormalization program in a scalar field theory where we assume Dirichlet-Dirichlet or Neumann-Neumann boundary conditions on rigid surfaces one has to introduce surface counterterms. To write the full renormalized action for the theory with rigid boundaries we need two regulators: the first one is the usual ϵ that is introduced in the dimensional regularization procedure and the second one which we call η , represents the distance to a boundary. Accordingly we will show that the full renormalized action must be given by:

$$S(\varphi) = \int_0^L dz \int d^{d-1}r \left(\frac{A(\epsilon)}{2} (\partial_\mu \varphi)^2 + \frac{B(\epsilon)}{2} \varphi^2 + \frac{C(\epsilon)}{4!} \varphi^4 \right)$$

$$\begin{aligned}
& + \int d^{d-1}r \left(c_1(\eta) \varphi^2(\vec{r}, 0) + c_2(\eta) \varphi^2(\vec{r}, L) \right) \\
& + \int d^{d-1}r \left(c_3(\eta) \varphi^4(\vec{r}, 0) + c_4(\eta) \varphi^4(\vec{r}, L) \right), \tag{18}
\end{aligned}$$

where $A(\epsilon)$, $B(\epsilon)$ and $C(\epsilon)$ are the usual coefficients for the bulk counterterms and the coefficients $c_i(\eta)$, $i = 1, \dots, 4$, which depend on the boundary conditions for the field, are the coefficients for the surface counterterms. As usual, all of these coefficients must be calculated order by order in perturbation theory. Note that we are interested in systems that are invariant under translation along directions parallel to the plates, which implies that the full momentum is not conserved. For such conditions, a more convenient representation for the n -point Schwinger functions to implement the perturbative renormalization is a mixed (\vec{p}, z) representation. Careless one-loop perturbation theory leads to ultraviolet counterterms that depend on the distance between the plates or also to the absence of surface counterterms [22] [23].

In a straightforward way, in the Matsubara formalism all the Feynman rules are the same as in the zero temperature case, except that the momentum-space integrals over the zeroth component is replaced by a sum over discrete frequencies. For the case of bosons fields we have to perform the replacement

$$\int \frac{d^d p}{(2\pi)^d} f(p) \rightarrow \frac{1}{\beta} \sum_n \int \frac{d^{d-1} p}{(2\pi)^{d-1}} f\left(\frac{2n\pi}{\beta}, \vec{p}\right), \tag{19}$$

where we are using the following notation: $(\int d^{d-1}r = \int_0^\beta dr_1 \int d^{d-2}r)$.

We begin the study of the interacting theory by building the one-loop correction $(G_1^{(2)}(\lambda_0, x, x'))$ to the bare two-point Schwinger function $G_0^{(2)}(x, x')$, for both the DD and NN boundary condi-

tions. Using the Feynman rules we have that $G_1^{(2)}(\lambda_0, \vec{r}_1, z_1, \vec{r}_2, z_2)$ can be written as

$$G_1^{(2)}(\lambda_0, \vec{r}_1, z_1, \vec{r}_2, z_2) = \frac{\lambda_0}{2} \int d^{d-1}r \int_0^L dz G_0^{(2)}(\vec{r}_1 - \vec{r}, z_1, z) G_0^{(2)}(\vec{0}, z) G_0^{(2)}(\vec{r} - \vec{r}_2, z, z_2). \quad (20)$$

Even though the functions $G_0^{(2)}(\vec{r}_1 - \vec{r}_2, z_1, z_2)$ and $G_0^{(2)}(\vec{r}_2 - \vec{r}_3, z_2, z_3)$ are singular at coincident points $(\vec{r}_1 = \vec{r}_2, z_1 = z_2)$ and $(\vec{r}_2 = \vec{r}_3, z_2 = z_3)$, the singularities are integrable for points outside the plates. It is worth mentioning that the most simple way to take into account the boundary is to implement the boundary conditions through the explicit form of the free two-point Schwinger function $G_0^{(2)}(x - y, m_0)$. A straightforward substitution yields the order λ_0 correction to the bare two-point Schwinger function in the one-loop approximation for the case of Dirichlet-Dirichlet boundary conditions. Using the Feynman rules, $G_2^{(4)}(\lambda_0, x_1, x_2, x_3, x_4)$, i.e., the $O(\lambda_0^2)$ correction to the bare one-loop four-point Schwinger function, is given by

$$\begin{aligned} G_2^{(4)}(\lambda_0, \vec{r}_1, z_1, \vec{r}_2, z_2, \vec{r}_3, z_3, \vec{r}_4, z_4) &= \frac{\lambda_0^2}{2} \int d^{d-1}r \int d^{d-1}r' \int_0^L dz \int_0^L dz' G_0^{(2)}(\vec{r}_1 - \vec{r}, z_1, z) \\ &\quad G_0^{(2)}(\vec{r}_2 - \vec{r}, z_2, z) \left(G_0^{(2)}(\vec{r} - \vec{r}', z, z') \right)^2 \\ &\quad G_0^{(2)}(\vec{r}' - \vec{r}_3, z', z_3) G_0^{(2)}(\vec{r}' - \vec{r}_4, z', z_4). \end{aligned} \quad (21)$$

Note that we suppress the m_0 term in each expression. Again, all G_0 's are singular at coincident points, but the singularities are integrable for points outside the plates, except for $G_0^{(2)}(\vec{r} - \vec{r}', z, z')$.

Having in mind the above discussion, in this section we will study the following expressions:

$$\frac{\lambda_0}{2} \int d^{d-1}r \int_0^L dz \left(G_0^{(2)}(\vec{0}, z) \right), \quad (22)$$

and

$$\frac{\lambda_0^2}{2} \int d^{d-1}r \int d^{d-1}r' \int_0^L dz \int_0^L dz' \left(G_0^{(2)}(\vec{r} - \vec{r}', z, z') \right)^2. \quad (23)$$

Let us first study $\frac{1}{2} G_0^{(2)}(\vec{0}, z) \equiv I(z, m_0, L, \beta, d)$, and define the following quantities: $\frac{1}{b} = \frac{2}{\beta}$, $L = a$ and finally the dimensionless coupling constant $g = \mu^{4-d} \lambda_0$. Therefore, the argument in the integral defined in Eq.(22), $I(z, m_0, a, b, d)$ can be written as

$$I(z, m_0, a, b, d) = \frac{g}{2(2\pi)^{d-2}ab} \sum_{n=-\infty}^{\infty} \sum_{n'=1}^{\infty} \sin^2\left(\frac{n'\pi z}{a}\right) \int d^{d-2}p \frac{1}{\left(\vec{p}^2 + \left(\frac{n'\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + m_0^2\right)}. \quad (24)$$

There are two points that we would like to stress. First to perform analytic regularizations we have to introduce a parameter μ with dimension of mass in order to have dimensionless quantities raised to a complex power. Second, the generalization for the case of Neumann boundary conditions is straightforward, although in this case infrared divergences associated with the $n = 0$ mode will appear in the case of massless scalar field. To circumvent this situation, we must have a finite Euclidean volume to regularize the model in the infrared, or trying to implement a resummation to generate a thermal mass. We will return to this point latter.

Using trigonometric identities, it is convenient to write the amputated one-loop two-point Schwinger in two parts. The first comprises the contributions that do not depend on the distance to the boundary, and the second the contributions that do depend on this distance. Therefore, the quantity $I(z, m_0, a, b, d)$ can be split in two parts $T_1(m_0, a, b, d)$ and $T_2(z, m_0, a, b, d)$, i.e.:

$$I(z, m_0, a, b, d) = T_1(m_0, a, b, d) + T_2(z, m_0, a, b, d). \quad (25)$$

The first quantity $T_1(m_0, a, b, d)$, independent on the distance to the boundaries can be expressed

in the following way:

$$T_1(m_0, a, b, d) = I_0(m_0, a, b, d) + I_1(m_0, a, b, d) + I_2(m_0, a, b, d), \quad (26)$$

where each term is given respectively by:

$$I_0(m_0, a, b, d) = -\frac{g}{16(2\pi)^{d-2}ab} \int d^{d-2}p \frac{1}{(\vec{p}^2 + m_0^2)}, \quad (27)$$

$$I_1(m_0, a, b, d) = \frac{g}{8(2\pi)^{d-2}ab} \sum_{n=1}^{\infty} \int d^{d-2}p \frac{1}{(\vec{p}^2 + m_0^2 + (\frac{n\pi}{a})^2)}, \quad (28)$$

and finally

$$I_2(m_0, a, b, d) = \frac{g}{4(2\pi)^{d-2}ab} \sum_{n,n'=1}^{\infty} \int d^{d-2}p \frac{1}{(\vec{p}^2 + (\frac{n\pi}{a})^2 + (\frac{n'\pi}{b})^2 + m_0^2)}. \quad (29)$$

The contribution that depends on the distance to the boundaries given by $T_2(z, m_0, a, b, d)$, can be split in the following way:

$$T_2(z, m_0, a, b, d) = I_3(z, m_0, b, d) + I_4(z, m_0, a, b, d) + I_5(z, m_0, b, d) + I_6(z, m_0, a, b, d). \quad (30)$$

Each term contributing to $T_2(z, m_0, a, b, d)$ is given, respectively by:

$$I_3(z, m_0, b, d) = \frac{g}{2b} h(d) \int_{m_0}^{\infty} dv (v^2 - m_0^2)^{\frac{d-4}{2}} \exp(-2vz), \quad (31)$$

$$I_4(z, m_0, a, b, d) = \frac{g}{2b} h(d) \int_{m_0}^{\infty} dv (v^2 - m_0^2)^{\frac{d-4}{2}} (\coth av - 1) \cosh 2vz, \quad (32)$$

$$I_5(z, m_0, b, d) = \frac{g}{b} h(d) \sum_{n=1}^{\infty} \int_{m_0}^{\infty} dv \left(v^2 - m_0^2 - \left(\frac{n\pi}{b} \right)^2 \right)^{\frac{d-4}{2}} \exp(-2vz), \quad (33)$$

and finally

$$I_6(z, m_0, a, b, d) = \frac{g}{b} h(d) \sum_{n=1}^{\infty} \int_{\alpha}^{\infty} dv \left(v^2 - m_0^2 - \left(\frac{n\pi}{b} \right)^2 \right)^{\frac{d-4}{2}} (\coth av - 1) \cosh 2vz. \quad (34)$$

In the above expression the quantity α is given by

$$\alpha = \left(m_0^2 + \left(\frac{n\pi}{b} \right)^2 \right)^{\frac{1}{2}}, \quad (35)$$

and $h(d)$, that appears in Eqs.(31), (32), (33) and (34) is an entire function given by

$$h(d) = \frac{1}{4(4\pi)^{\frac{d-2}{2}}} \frac{1}{\Gamma(\frac{d-2}{2})}. \quad (36)$$

Let us investigate each contribution in detail. Using dimensional regularization we obtain for $I_0(m_0, d)$ the following expression:

$$I_0(m_0, a, b, d) = -\frac{g}{16 ab (2\sqrt{\pi})^{d-2}} \Gamma(2 - \frac{d}{2}) (m_0^2)^{\frac{d}{2}-2}. \quad (37)$$

An analytic expression for the Gamma function $\Gamma(z)$, defined in the whole complex plane, can be found and in the neighborhood of a pole $z = -n$, ($n = 0, 1, 2, \dots$) the Gamma function has the representation

$$\Gamma(z) = \frac{(-1)^n}{n!} \frac{1}{(z+n)} + \Omega(z+n), \quad (38)$$

with regular part $\Omega(z+n)$. Using that $4-d = \epsilon$ and the duplication formula for the Gamma function $\Gamma(z)$ we have

$$I_0(m_0, a, b, d)|_{d=4} = -\frac{g}{16\pi ab m_0^\epsilon} \left(\frac{1}{\epsilon} + \Omega(\epsilon) \right). \quad (39)$$

Here one may adopt different renormalization schemes. We can choose the minimal subtraction (MS) scheme, in which we eliminate only the pole term $\frac{1}{\epsilon}$ in the dimensionally regularized expression for the Schwinger functions. Another choice is the modified minimal subtraction ($\overline{\text{MS}}$) scheme,

where we eliminate not only the pole term $\frac{1}{\epsilon}$ but also the regular part around the pole. Note that in the minimal subtraction scheme the counterterms acquire the simplest expression, while the renormalized Schwinger functions have more complicated expressions. Let us analyse the second expression, given by $I_1(m_0, a, b, d)$. Using dimensional regularization it is possible to show that

$$I_1(m_0, a, b, d) = \frac{g}{8(2\sqrt{\pi})^{d-2}ab} \Gamma(2 - \frac{d}{2}) \sum_{n=1}^{\infty} \frac{1}{(m_0^2 + (\frac{n\pi}{a})^2)^{2-\frac{d}{2}}}. \quad (40)$$

We note that to extract a finite result from $I_1(m_0, a, b, d)$ we still have to use the analytic extension of the Epstein-Hurwitz zeta function. A direct calculation gives

$$I_1(m_0, a, b, d) = -\frac{g}{8ab} m_0^{d-4} \frac{\sqrt{\pi}}{(2\sqrt{\pi})^{d-1}} \Gamma(2 - \frac{d}{2}) + \frac{g m_0^{d-3}}{8b} \frac{1}{(2\pi)^{d-1}} \left(\Gamma\left(\frac{3-d}{2}\right) + 4 \sum_{n=1}^{\infty} (am_0 n)^{\frac{3-d}{2}} K_{\frac{3-d}{2}}(2m_0 na) \right). \quad (41)$$

The first term in the above equation is a polar part and the second one is finite. Assuming the minimal subtraction scheme, $I_1(m_0, a, b, d)$ becomes finite. The next term that we have to analyse is $I_2(m_0, a, b, d)$ defined by:

$$I_2(m_0, a, b, d) = \frac{g}{4ab} \frac{1}{(2\pi)^{d-2}} \sum_{n, n'=1}^{\infty} \int d^{d-2}p \frac{1}{(\vec{p}^2 + (\frac{n\pi}{a})^2 + (\frac{n'\pi}{b})^2 + m_0^2)}. \quad (42)$$

The contribution given by the above equation is a part of the amputated one-loop two-point Schwinger function that does not depend on the distance to the boundaries, and in the renormalization procedure it will require only a usual bulk counterterm. The form of the counterterm is given by the principal part of the Laurent expansion of Eq.(42) around some d , which must be given by the analytic extension of the Epstein zeta function in the complex d plane. The structure

of the divergences of the Epstein zeta function is well known in the literature [30] [31] [32] [33]. Since the polar structure of the above equation can be found in the literature, we will focus only on the position-dependent divergent part given by $T_2(z, m_0, a, b, d)$. We are now in position to discuss the behavior of $I_3(z, m_0, b, d)$, $I_4(z, m_0, a, b, d)$, $I_5(z, m_0, b, d)$ and finally $I_6(z, m_0, a, b, d)$.

Let us first analyse $I_3(z, m_0, b, d)$. Using the following integral representation of the modified Bessel functions of third kind, or Macdonald's functions $K_\nu(x)$ [34],

$$\int_u^\infty (x^2 - u^2)^{\nu-1} e^{-\mu x} dx = \frac{1}{\sqrt{\pi}} \left(\frac{2u}{\mu}\right)^{\nu-\frac{1}{2}} \Gamma(\nu) K_{\nu-\frac{1}{2}}(u\mu), \quad (43)$$

which is valid for $u > 0$, $\text{Re}(\mu) > 0$ and $\text{Re}(\nu) > 0$, we see that $I_3(z, m_0, a, b, d)$ can be written in terms of these functions. A simple substitution gives

$$I_3(z, m_0, a, b, d) = \frac{2}{b} \frac{h(d)}{(2\sqrt{\pi})^{d-1}} \left(\frac{m_0}{z}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(2m_0 z). \quad (44)$$

Using a asymptotic formula for the Bessel function, $I_3(z, m_0, a, b, d)$ is given by

$$I_3(z, m_0, a, b, d) = \frac{2}{b} \frac{h(d)}{(2\sqrt{\pi})^{d-1}} \frac{\Gamma(\frac{d-3}{2})}{z^{d-3}}. \quad (45)$$

We can see that we have a divergent behavior as $z \rightarrow 0$, which demands a surface counterterm. Let us show that the other terms also contain surface divergences, and study $I_4(z, m_0, a, b, d)$. To advance in the calculations, we have to extend the binomial series for both positive or negative integral exponents, written in the form

$$(1+x)^k = \sum_{n=0}^{\infty} C_n^k x^n. \quad (46)$$

First, it is possible to show that the binomial expansion holds for any real exponent α , $|x| < 1$ and $\alpha \in R$, i.e.,

$$(1+x)^\alpha = \sum_{n=0}^{\infty} C_\alpha^n x^n, \quad (47)$$

where C_α^n are the generalization of the binomial coefficients. Since we are using dimensional regularization, it is possible to extend the binomial expansion when both the exponent α as well the variable x assume complex values. For this purpose we use the following theorem:

For any complex exponent α and any complex z in $|z| < 1$, the binomial series

$$\sum_{n=0}^{\infty} C_\alpha^n z^n = 1 + C_\alpha^1 z + \dots + C_\alpha^n z^n + \dots \quad (48)$$

converges and has for sum the principal value of the power $(1+z)^\alpha$, where the principal value of the power b^a is given by the number uniquely defined by the formula $b^a = \exp(a \ln b)$, where a and b denotes any complex numbers, with $b \neq 0$ as the only condition, and $\ln b$ is given its principal value. Going back to $I_4(z, m_0, a, b, d)$, using the generalization of the binomial theorem, let us define $C^{(1)}(d, k) = \frac{1}{2} h(d) (-1)^k C_{\frac{d-4}{2}}^k$ to obtain

$$I_4(z, m_0, a, b, d) = \frac{g}{a^{d-3} b} \sum_{k=0}^{\infty} C^{(1)}(d, k) (m_0 a)^{2k} \int_{m_0 a}^{\infty} u^{d-4-2k} (\coth u - 1) \cosh\left(\frac{2uz}{a}\right). \quad (49)$$

Let us use the following integral representation of the Gamma function,

$$\int_0^{\infty} dt t^{\mu-1} e^{-\nu t} = \frac{1}{\nu^\mu} \Gamma(\mu), \quad \text{Re}(\mu) > 0, \quad \text{Re}(\nu) > 0, \quad (50)$$

and also the following integral representation of the product of the Gamma function times the

Hurwitz zeta function

$$\int_0^\infty dt t^{\mu-1} e^{-\alpha t} (\coth t - 1) = 2^{1-\mu} \Gamma(\mu) \zeta(\mu, \frac{\alpha}{2} + 1) \quad \text{Re}(\alpha) > 0, \quad \text{Re}(\mu) > 1, \quad (51)$$

where $\zeta(s, u)$ is the Hurwitz zeta function defined by [34]

$$\zeta(s, u) = \sum_{n=0}^{\infty} \frac{1}{(n+u)^s}, \quad \text{Re}(s) > 1 \quad u \neq 0, -1, -2, \dots \quad (52)$$

It is not difficult to show that $I_4(z, m_0, a, b, d)$ contains surface divergences at $z = 0$ and also $z = a$. For more details, see for example Ref. [35]. The other expression that we have to study is $I_5(z, m_0, a, b, d)$. Using an integral representation of the Bessel function of third kind we have:

$$I_5(z, m_0, a, b, d) = \frac{1}{b} \frac{1}{(2\sqrt{\pi})^{d-1}} \sum_{n=1}^{\infty} \left(\frac{\alpha}{z}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(2\alpha z). \quad (53)$$

Using an asymptotic representation of the Bessel function it is possible to present also the singular behavior near $z = 0$. Let us finally investigate $I_6(z, m_0, a, b, d)$. A simple calculation for the massless case gives

$$I_6(z, m_0, a, b, d)|_{m=0} = \frac{1}{a^{d-3}b} \sum_{k=0}^{\infty} C^{(2)}(d, k) \left(\frac{a}{b}\right)^{2k} \sum_{n=1}^{\infty} n^{2k} \int_{\frac{n\pi a}{b}}^{\infty} du u^{d-4-2k} (\coth u - 1) \cosh\left(\frac{2uz}{a}\right), \quad (54)$$

where $C^{(2)}(d, k) = h(d)(-1)^k C_{\frac{d-4}{2}}^k \pi^{2k}$ is an entire function in the complex d plane. The integral that appear in Eq.(54) cannot be evaluated explicitly in terms of well known functions. Nevertheless it is possible to write Eq.(54) in a convenient way where the structure of the divergences near the plate when $y \rightarrow b$ appear. Clearly for details see Ref. [35]. In the next section we will investigate the singularities of the four-point Schwinger function.

4 The four-point Schwinger function in the one-loop approximation

We now turn our attention to the four-point Schwinger function in the one-loop approximation. For simplicity we shall study only the zero temperature case. In this section we are following the discussion developed in Ref. [25]. Introducing new variables as $u_{\pm} \equiv z \pm z'$, and also $(\vec{\rho} = \vec{r} - \vec{r}')$, the zero-temperature two-point Schwinger function in the tree-level $G_0^{(2)}(\vec{\rho}, z, z')$ can be split into

$$G_0^{(2)}(\vec{\rho}, z, z') = G_+^{(2)}(\vec{\rho}, u_+) + G_-^{(2)}(\vec{\rho}, u_-), \quad (55)$$

where we are defining $A_n(a, m_0, d, \vec{\rho})$ by

$$A_n(a, m_0, d, \vec{\rho}) = \frac{1}{(2\pi)^{d-1}} \int d^{d-1}p \frac{e^{i\vec{p} \cdot \vec{\rho}}}{(\vec{p}^2 + (\frac{n\pi}{a})^2 + m_0^2)}, \quad (56)$$

and so $G_{\pm}^{(2)}(\vec{\rho}, u_{\pm})$ can be expressed as

$$G_{\pm}^{(2)}(\vec{\rho}, u_{\pm}) = \mp \frac{1}{a} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi u_{\pm}}{a}\right) A_n(a, m_0, d, \vec{\rho}). \quad (57)$$

Before proceeding, let us present an explicit formula for the free two-point Schwinger function $G_{\pm}^{(2)}(\rho, u_{\pm})$ in terms of Bessel functions. Let us define an analytic function $f(d)$ by

$$f(d) = \frac{1}{\sqrt{\pi}(2\pi)^{\frac{d-1}{2}}} \frac{\Gamma(\frac{d-2}{2})}{\Gamma(\frac{d-3}{2})}. \quad (58)$$

Strictly speaking, it is possible to show that we can write $G_{\pm}^{(2)}(\rho, u_{\pm})$ in terms of the Bessel function of third kind. To this end, we use the standard formula

$$\frac{1}{(2\pi)^d} \int d^d r F(r) e^{i\vec{k} \cdot \vec{r}} = \frac{1}{\sqrt{\pi}(2\pi)^{\frac{d}{2}}} \frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})} \int_0^{\infty} F(r) r^{\frac{d}{2}} J_{\frac{d-3}{2}}(kr) dr, \quad (59)$$

which leads us to:

$$G_{\pm}^{(2)}(\rho, u_{\pm}) = \mp \frac{f(d)}{\rho^{\frac{d-3}{2}} a} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi u_{\pm}}{a}\right) \int_0^{\infty} dp \frac{p^{\frac{d-1}{2}}}{(p^2 + (\frac{n\pi}{L})^2 + m_0^2)} J_{\frac{d-3}{2}}(p\rho), \quad (60)$$

where $J_{\nu}(x)$ is the Bessel function of the first kind of order ν . The integral in Eq.(60) can be calculated by using the result [34]

$$\int_0^{\infty} dx \frac{x^{\nu+1} J_{\nu}(ax)}{(x^2 + b^2)} = b^{\nu} K_{\nu}(ab), \quad (61)$$

implying that it is possible to write $G_{\pm}^{(2)}(\rho, u_{\pm})$ as

$$G_{\pm}^{(2)}(\rho, u_{\pm}) = \mp \frac{f(d)}{\rho^{\frac{d-3}{2}} a} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi u_{\pm}}{a}\right) \left(\left(\frac{n\pi}{a}\right)^2 + m_0^2\right)^{\frac{d-3}{4}} K_{\frac{d-3}{2}}\left(\rho \sqrt{m_0^2 + \left(\frac{n\pi}{a}\right)^2}\right). \quad (62)$$

Using Eq.(55) and the above formula, the explicit expression for the two-point Schwinger function in a generic d -dimensional Euclidean space confined between two flat parallel hyperplanes, where we assume Dirichlet-Dirichlet boundary conditions is given. It is difficult to use the above expressions for $G_{\pm}^{(2)}(\rho, u_{\pm})$ to investigate the analytic structure of the four-point function for both the bulk and near the boundaries. Nevertheless, it is clear that the divergences of the four-point function in the one-loop approximation appear at coincident points and therefore the singular behavior is encoded in the polar part of $M(\lambda_0, a, m, d)$ given by

$$M(\lambda_0, a, m_0, d) = g^2 \int d^{d-1}r \int d^{d-1}r' \int_0^a dz \int_0^a dz' F(\vec{r}, \vec{r}', z, z') \left(G_0^{(2)}(\vec{r} - \vec{r}', z, z')\right)^2. \quad (63)$$

It is easy to show that $G_2^{(4)}(\lambda_0, a, m_0, d)_{amp}$ is given by

$$G_2^{(4)}(\lambda_0, a, m_0, d)_{amp} = \frac{g^2}{2(2\pi)^{2d-2}} \int d^{d-1}r \int d^{d-1}r' \int d^{d-1}k \int d^{d-1}q \sum_{n=1}^{\infty} \frac{e^{i\vec{p} \cdot (\vec{q} - \vec{k})}}{(\vec{q}^2 + (\frac{n\pi}{a})^2 + m_0^2)(\vec{k}^2 + (\frac{n\pi}{a})^2 + m_0^2)}, \quad (64)$$

where $F(\vec{r}, \vec{r}', z, z')$ is a regular function. As with the one-loop two-point function, it is not difficult to realize that the above equation has two kinds of singularities, those coming from the bulk and those arising from the behavior near the surface. As before, the behavior in the bulk is similar to the thermal field theory case and consequently we will discuss only the singularities arising from the boundaries. This can be done studying the polar part of $\tilde{M}(\lambda_0, a, m_0, d)$ given by

$$\tilde{M}(\lambda, a, m_0, d) = \frac{g^2}{2} \int_0^a dz \int_0^a dz' \mathcal{F}(z, z') \left(G_0^{(2)}(\vec{0}, z, z') \right)^2, \quad (65)$$

where $\mathcal{F}(z, z')$ is a regular function. Now, we recall that the form of $G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0}$ is given by,

$$G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0} = \mp \frac{1}{(2\pi)^{d-1}a} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi u_{\pm}}{a}\right) \int d^{d-1}p \frac{1}{\left(\vec{p}^2 + m_0^2 + \left(\frac{n\pi}{a}\right)^2\right)}, \quad (66)$$

where it is not difficult to show that

$$G_{\pm}^{(2)}(\rho, u_{\pm})|_{\rho=0} = \mp \left(-\frac{1}{2a} A_0(\rho, L, m_0)|_{\rho=0} + f_2(a, m_0, d, \frac{u_{\pm}}{2}) \right). \quad (67)$$

In the above definition we are making use of the auxiliary function $f_2(a, d, m_0, z)$ given by

$$f_2(a, m_0, d, z) = \frac{1}{2(2\pi)^{d-1}} \int d^{d-1}p \frac{1}{\sqrt{\vec{p}^2 + m_0^2}} \frac{\cosh((a-2z)\sqrt{\vec{p}^2 + m_0^2})}{\sinh(a\sqrt{\vec{p}^2 + m_0^2})}. \quad (68)$$

Note that the amputated one-loop two-point Schwinger function can be decomposed in a translational invariant part and a translational invariance breaking part, given exactly by $f_2(a, m_0, d, z)$.

When we sum to find the free propagator, we end up with the following expression

$$G_0^{(2)}(\rho, z, z')|_{\rho=0} = f_2(a, m_0, d, \frac{u_-}{2}) - f_2(a, m_0, d, \frac{u_+}{2}). \quad (69)$$

For the sake of simplicity, we will discuss only the massless case once the singularities of the massive case have the same structure as in the massless one. The function $f_2(a, m_0, d, \frac{u_+}{2})$ is non-singular in the bulk, i.e., in the interior of the interval $[0, a]$, while $f_2(a, m_0, d, \frac{u_-}{2})$ has a singularity along the line $z = z'$. Indeed, closer inspection shows that for $0 \leq z, z' \leq a$ the only singularities are those at $u_+ = 0$, $u_+ = 2a$ and also $u_- = 0$. The former two are genuinely boundary singularities (the two conditions imply $z, z' \rightarrow 0$ or $z, z' \rightarrow a$), while the last comes from $z = z'$ in the whole domain and is just the standard bulk singularity. In fact, using the structure of the two-point function and showing just those terms from which singularities might arise, one finds that the counterterms for \tilde{M} are given by

$$-\text{pole} \int_0^a dz \int_0^a dz' \left[\frac{C_1}{(z + z')^{d-2}} + \frac{C_2}{(2a - z - z')^{d-2}} + \frac{C_3}{(z - z')^{d-2}} + \dots \right]^2, \quad (70)$$

where $C_i, i = 1, \dots, 3$ are regular functions that do not depend on z or z' . From this discussion it is clear that in order to render the field theory finite, we must introduce surface terms in the action. This is a general statement. For any fields that satisfy boundary conditions that break the translational invariance it suffices to introduce surface counterterms in the action, in addition to the usual bulk counterterms, to render the theory finite in the ultraviolet [36] [37] [38]. Now we are able to discuss whether in the Casimir configuration the infrared problems can be solved for the case of Neumann boundary conditions. For the case of massless $(\lambda\varphi^4)_d$ theory at finite temperature, the infrared problem can be solved after a resummation procedure [17] [18] [19] [20] [39]. The key point for the solution of the infrared problem is to use the Dyson-Schwinger equation to rewrite the self-energy gap equation. Simple inspection of Eq.(24) show us that it is not possible

to implement such scheme in a situation where there is a break of translational invariance.

A different possibility to approach the infrared problem is to single out the zero mode component of the field, treating the non-zero modes perturbatively and treating the zero mode exactly. This is a standard procedure in high-temperature field theory, where by means of the dimensional reduction idea, we relate the thermal Schwinger functions in a d -dimensional Euclidean space to zero temperature Schwinger functions in a $(d - 1)$ dimensional Euclidean space [40] [41] [42]. In this situation we have a dimensionally reduced effective theory. The key point in this construction is the fact that the leading infrared behavior of any field theory at high temperature in a d -dimensional Euclidean space is governed by the zero frequency Matsubara mode.

5 Discussions and conclusions

In this paper we were interested in the analysis of the important questions of perturbative expansion and renormalization program in quantum field theory with boundary conditions that break translation symmetry, assuming that the system is in equilibrium with a reservoir at temperature β^{-1} . Specifically, the purpose of this paper is to study the renormalization procedure up to one-loop level in the $(\lambda\varphi^4)_d$ theory at finite temperature assuming that the scalar field satisfies Dirichlet-Dirichlet or Neumann-Neumann boundary conditions on two parallel hyperplates.

We first obtained the regularized one-loop diagrams associated with scalar field defined in the Casimir configuration in a d -dimensional Euclidean space. We obtained a well-know result concerning surface divergences that appear in the one-loop two-point and four-point Schwinger

functions as a consequence of the uncertainty principle. There are at least three different possible solutions to eliminate these divergences. The first one is to take into account that real materials have imperfect conductivity at high frequencies. As was stressed by many authors, the infinities that appear in renormalized values of local observables for the ideal conductor (or perfect mirror) represent a breakdown of the perfect-conductor approximation. A wavelength cutoff corresponding to the finite plasma frequency must be included. The second one is to substitute classical boundary conditions by classical potentials; for previous papers using this idea see for example [43] [44] [45]. A localized boundary with some cut-off can also be used to replace the potential. Nevertheless, it is necessary to renormalize the potential [25]. The third one regards a quantum mechanical treatment of the boundary conditions. A fruitful approach to avoid surface divergences, discussed by Kennedy et al. [46] is to treat the boundary as a quantum mechanical object. This approach was developed by Ford and Svaiter [47] to produce finite values for the renormalized $\langle \varphi^2 \rangle$ and other quantities that diverge as one approaches the classical boundary.

Consequently, we have two main distinct directions for future investigations. The first is related to the infrared divergences of our model. Infrared divergences of massless thermal field theory arise from the zero frequency Matsubara modes, so we construct an effective $(d - 1)$ dimensional theory by integrating out the nonstatic modes and therefore the zero frequency Matsubara modes which are responsible for infrared divergences can be treated separately. The second direction is related to the surface divergences. In the Euclidean formalism for field theory, one may imagine that our simplified model of rigid boundaries is a good approximation only for points in the bulk; for

points close to the surfaces however, our approximation is no longer accurate and a model taking into account at least thermal fluctuations of the boundaries must be developed [48]. In other words, a fundamental understanding of the perturbative renormalization algorithm in the standard weak-coupling perturbative expansion of an Euclidean field in the presence of fluctuating boundaries is desired. This interesting situation of thermal fluctuating boundaries is under the investigation by the authors.

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